JAF-003-001308 **Seat No.** ________

B. Sc. (Sem. III) Examination

November - 2019

Mathematics : Paper BSMT-301(A)

(Theory)

(Linear Algebra, Calculus & Theory of Equations)

[Old Course]

Faculty Code : 003

Subject Code : 001308

Time : **2.30** Hours] [Total Marks : **70**

Instructions :

(1) **All** the questions are **compulsory**.

(2) Numbers given to the right indicate full marks of the questions.

1 Answer the following questions in short : **20**

(1) Define Linear dependence.

(2) Define Improper subspace of a vector space.

(3) Define Linear transformation.

(4) If $\dim U = 2$, $\dim V = 3$, then find the dim $L(U, V)$.

(5) Define Idempotent Linear Transformation.

(6) Check whether the set $A = \{(2,0,0), (0,0,1), (1,0,0)\}$ is LI or LD ?

(7) Define Dual of vector space.

(8) Define Basis of a vector space.

(9) Is the series $\sum \frac{1}{n^{\frac{1}{2}}}$ convergent ?

(10) The series $1+2+3+4+..........$ is divergent. (True/False)

(11) Is the series $\sum \frac{n^2+5}{3n^2+4}$ convergent ?

(12) Regula Falsi method is also known as _____.

(13) Write an example of transcendental equation.

(14) The curvature for the circle of radius 4 is _______.

(15) Write a formula to find curvature in Cartesian coordinates.

(16) Find the radius of curvature for the curve $s = 4a \sin \psi$.

(17) Find the equation whose roots are 10 times the roots of $2x^3 + 4x^2 - 3x + 16 = 0$.

(18) Define Double point.

(19) Define Singular point.

(20) Define Point of inflexion.

2 (a) Answer any **three** : **6**

(1) Find N_T and $n(T)$ for the linear transformation $T : R^3 \to R^2$ defined as $T(x, y, z) = (x - y + z, x + y - z), \forall (x, y, z) \in R^3$.

(2) Discuss the convergence of series $\sum \frac{1}{n^2} \sin \frac{1}{n}$.

(3) Discuss the convergence of series $\sum \frac{n^2}{3^n}$.

(4) Check whether V is vector space, where $V = \{(x, y) : x, y \in R\}$ and the operations are defined as for $(x_1, y_1), (x_2, y_2) \in V, (x_1, y_1) + (x_2, y_2) = (x_1, y_1)$ and for $\alpha \in R$ $\alpha(x_1, y_1) = (\alpha x_1, \alpha y_1)$.

(5) Check whether the sub set $\{(1,1,-1), (1,0,1), (1,1,0)\}$ of vector space R^3 is LD or LI.

(6) Find the eigen value of the linear transformation $T : R^2 \to R^2$, $T(a, b) = (3b, 2a - b)$, $\forall (a, b) \in R^2$ by considering the standard basis of R^2.

(b) Answer any **three** : **9**

(1) Prove that the intersection of two subspace of a vector space is also subspace.

(2) Let $T : R^2 \to R^2, T(x, y) = (x, -y); \forall (x, y) \in R^2$ and let $B_1 = \{(1,0), (0,1)\}$ and $B_2 = \{(1,1), (1,-1)\}$ be two basis of R^2. Find $[T; B_1, B_2]$.

(3) Extend L.I. set $A = \{1 - x + x^2, 2x - x^2 + x^3\}$ of vector space $P_3(R)$ to form a basis of $P_3(R)$.

(4) Let $T : V \to V$ be any linear transformation such that $T^2 - T + I = 0$ then prove that T is non-singular.

(5) Discuss the convergence of series

$$1 + \frac{1}{2} + \frac{1 \cdot 3}{2 \cdot 4} + \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6} + \frac{1 \cdot 3 \cdot 5 \cdot 7}{2 \cdot 4 \cdot 6 \cdot 8} + \ldots\ldots.$$

(6) Discuss the convergence of series

$$\frac{1}{1+2} + \frac{2}{1+2^2} + \frac{3}{1+2^3} + \frac{4}{1+2^4} + \ldots\ldots\ldots\ldots .$$

(c) Answer any **two** : **10**

(1) State and prove Rank - Nullity Theorem.

(2) Let $T : V \to V$ be a linear transformation and let B be any basis of V. Then prove that T is singular if and only if $\det([T; B]) = 0$.

(3) Prove that the set $A = \{(1,2,1), (2,1,0), (1,-1,2)\}$ forms a basis for R^3. Find coordinate of $(1,1,-1)$ with respect to this basis.

(4) If W_1 and W_2 are two subspace of finite dimensional vector space V, then prove that

$\dim(W_1 + W_2) = \dim W_1 + \dim W_2 - \dim(W_1 \cap W_2)$.

(5) Find radius of convergence and interval of convergence of the series $\sum \frac{(-3)^n x^n}{\sqrt{n+1}}$.

3 (a) Answer any **three** : **6**

(1) If $P(x) = x^4 - 5x^3 - 2x + 7$ then find $P''(-2)$.

(2) Explain bisection method to find real root of an equation.

(3) Show that the curve $y = x^4$ is concave upwards at the origin.

(4) Find asymptotes of the curve $x^2y^2 = a^2(x^2 + y^2)$ parallel to coordinate axes.

(5) Find radius of curvature of $y = c\cosh\frac{x}{c}$.

(6) For the curve $(x^2 + y^2)x - ay^2 = 0$, prove that origin is a cusp.

(b) Answer any **three** : **9**

(1) Find all the asymptotes of the curve

$4x^3 - 3xy^2 - y^3 + 2x^2 - xy - y^2 = 1 \cdot$

(2) Derive the formula to find $\frac{1}{\sqrt{N}}$ using Newton-Raphson method.

(3) Explain false position method to find real root of equation.

(4) Find double point of the curve

$x^3 + y^3 - 3x^2 - 3xy + 3x + 3y - 1 = 0$.

(5) Show that the parabola $y^2 = 4ax$ has no asymptotes.

(6) Show that origin is a point of inflection for the curve

$y = x^3, x \in \mathbb{R}$.

(c) Answer any **two** : **10**

(1) Find an approximate root of the equation

$x^3 + 2x^2 + 10x - 20 = 0$ by Newton-Raphson method.

(2) Using Horner's method, find the root of equation

$x^3 + 9x^2 - 18 = 0$ correct up to two decimal places.

(3) State Newton's method and using it find radius of curvature at origin for the curve $x^3 + y^3 = 3axy$.

(4) Show that radius of curvature of any point on the cardiod $r = a(1+\cos\theta)$ is $\frac{2}{3}\sqrt{2ar}$. Hence prove that $\frac{\rho^2}{r}$ is constant.

(5) Find the position and nature of double points of the curve $x^4 - 2ay^3 - 3a^2y^2 - 2a^2x^2 + a^4 = 0$.
